Question 1

(a) What is a data type? Define carefully the mathematical concepts of a
(i) *many sorted signature* Σ, and
(ii) *many sorted* Σ *algebra* A.
What aspects of a data type do these two mathematical concepts model?

(**7 marks**)

(b) What is an axiomatic specification for a data type and what is its purpose?

Give an account of each of the following axiomatic specifications (Σ, T) and their data types:
(i) Dedekind's axioms for the natural numbers; and
(ii) Commutative ring axioms for the integers.

What, if any, finite algebras satisfy these specifications?

(**14 marks**)

(c) Give an axiomatic specification for a general storage structure for data. Show how a conventional data structure (such as the array, record, stack, list and queue) can be derived from your general storage specification.

(**4 marks**)

Question 2

(a) Let A and B be Σ-algebras. Define carefully the concept of a
(i) Σ-*homomorphism* ϕ: A $\rightarrow$ B, and
(ii) Σ-*isomorphism* ϕ: A $\rightarrow$ B.
What are these concepts used for in the theory of abstract data types?

(**5 marks**)

(b) Let **R** be the set of integers and let (**R**, 0, +) be the algebra of real addition. Which of the following functions f: (**R**, 0, +) $\rightarrow$ (**R**, 0, +) is a homomorphism? Give reasons for your answer.

(i) f(x) = 10x
(ii) f(x) = 10x +1
(iii) f(x) = 1
(iv) f(x) = 0

(**4 marks**)

(c) Consider the following signature Σ_{Class} which models the idea of a class interface in object-oriented programming. The class given has two methods: *commands* that do not return data but simply change state, and *queries* that change state and also return data.

signature Class;
sorts state, data;
operations commandmeth: state $\times$ data $\rightarrow$ state;
querymeth: state $\times$ data $\rightarrow$ state
queryreturn: state $\times$ data $\rightarrow$ data
endsig

Turn Over

We suppose any Σ_{Class} algebra models of an object of this class. Let O and P be two Σ_{Class} algebras modelling two objects of the class Σ_{Class}. Let

$$\theta=(\theta_{state}, \theta_{data}): O \to P$$

be a Σ_{Class} homomorphism comparing O and P. Write down the *three* homomorphism equations for θ.

(**6 marks**)

(d) What is the difference between digital and analogue data? Show how to define the idea of a *digital data type* using a homomorphism.

(**10 marks**)

Question 3

(a) What is a *formal language* L? Define the *recognition problem* for the language L. Define the mathematical concept of a *grammar G* and how it defines a formal language L(G). Define the concept of a *context-free grammar.* Give an upper bound on the complexity of the *recognition problem* for context-free languages.

(**8 marks**)

(b) Describe the basic steps in defining syntax by means of a *modular context-free grammars*. What are the strengths and weaknesses of the method?

Illustrate your answer using a case study chosen from *one* of the following:

(i) postal addresses for a country of your choice;
(ii) file name formats for an operating system of your choice;
(iii) addresses for a type of URL of your choice.

(**7 marks**)

(c) Discuss the idea of a *kernel language* and its *extensions*. Use a case study to illustrate the definition of the *syntax* and *semantics* of such languages. Obvious examples of languages to extend are either (i) the language of all signatures, which is an interface definition language for data types; or (ii) the language for **while** programs, which is a simple imperative language.

(**10 marks**)

End