CS 132
Algorithms and Computation
(Answer 2 questions out of 3)

Question 1

Consider the following algorithm, which takes two natural numbers a and b as input:

SPHINX (a, b)

```
1  d ← 0
2  i ← a
3  while i > 0 do
4        d ← d + b
5        i ← i − 1
6  od
7  output d
```

a) Execute the algorithm SPHINX for the inputs

i) $a = 0,\ b = 3$

ii) $a = 1,\ b = 3$

iii) $a = 2,\ b = 3$

Use for this the following scheme:

- $a = \ldots$
- $b = \ldots$
-

PC	d	i	comments

- The output is ...

[6 marks]

b) Formulate the computational problem which is solved by SPHINX.

[4 marks]

c) Argue why SPHINX terminates for all inputs.

[5 marks]

...

d) Consider the running time of SPHINX, where the assignments and comparisons have cost 1, and all other operations are free (i.e. the execution of the lines 6 & 7 does not incur any cost).

i) Give the total number of steps for the inputs

- $a = 0$,
- $a = 1$, and
- $a = 2$.

(Note that the running time does not depend on the input b in our cost model.)

[6 marks]

ii) Give a formula for the running time depending on a.

[4 marks]

Question 2

a) Consider the following list of natural numbers: $4, 3, 2, 1$. Explain the Divide & Conquer approach of the algorithm MergeSort on this concrete example

[8 marks]

b) Prove the following:

i) $4n^2 - 3n + 6 = O(n^2)$

ii) $n^2 + 7 \neq O(n)$

iii) $2n^2 - 4n + 5 = \Omega(n^2)$

iv) $3n - 4 \neq \Omega(n^2)$

[8 marks]

c) Consider the following description of integer constants taken from the *Java Language Specification*:

- A `NonZeroDigit` is an element of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
- A `Digit` is an element of the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
- A `DecimalNumeral` is a non-empty word. If the first symbol of a `DecimalNumeral` is '0', the word has length 1. If the first symbol of a `DecimalNumeral` is a `NonZeroDigit`, the word can have arbitrary length, where all symbols apart from the first are out of the set `Digit`.

Write regular expressions over the signature $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ that describe the languages

i) `NonZeroDigit`

ii) `Digit`

iii) `DecimalNumeral`

[9 marks]

Question 3

a) Consider the operation of *concatenation*, denoted by the symbol $\circ$, of two strings.

i) Give the definition of concatenation.

[4 marks]

ii) Let $\Sigma = \{a, b, c\}$ be an alphabet. Prove each of the following:

i. $abc = ab \circ c$

ii. $abdd \neq ab \circ cd$

[4 marks]

b) Let $L := \{w \in \{0,1\}^* \mid w$ has an even number of $0's$ and an even number of $1's\}$ be a language over the alphabet $\Sigma = \{0, 1\}$.

i) Give an example of a word which belongs to L, and an example of a word which does not belong to L.

ii) Draw the graph of a deterministic, finite automaton which recognises L.

[9 marks]

c) Let L be the set of strings of 0's and 1's such that there are two 0's separated by a positive number of positions that is a multiple of 3. (Note that 0 is not an allowable multiple of 3.)

i) Give an example of a word which belongs to L, and an example of a word which does not belong to L.

ii) Draw the graph of a non-deterministic, finite automaton which recognises L.

[8 marks]