

CS_342 2004/05
Constraint Satisfaction Problems and Applications

(Attempt 2 questions out of 3)

Question 1 SAT algorithms

- (a) Discuss the trivial SAT algorithm (enumerating all total assignments), what it can achieve and how it can be improved. **[10 marks]**
- (b) DLL-like SAT algorithms
- (i) Explain the improved DLL algorithm scheme and the possible choices made in the computation. **[5 marks]**
- (ii) Decide satisfiability of the clause-sets F_1, F_2 by using the improved DLL algorithm, where

$$F_1 := \{ \{c, b, \bar{e}\}, \{\bar{c}, d, a\}, \{\bar{b}, g\}, \{\bar{d}, f\}, \{c, \bar{b}, \bar{g}\}, \{\bar{c}, \bar{d}, \bar{f}\}, \\ \{\bar{d}, f\}, \{d, \bar{a}\}, \{b, e\} \},$$

while F_2 is given in DIMACS format as

```
p cnf 5 15
-1 -3 4 0
-3 4 5 0
-2 -3 4 0
-1 -4 -5 0
-2 -4 -5 0
2 -4 5 0
-1 -2 3 0
2 3 4 0
3 4 -5 0
1 -3 -5 0
2 -3 4 0
-2 -3 -4 0
-3 4 5 0
2 -3 5 0
1 -2 -3 0
```

Show the backtracking trees and explain all your choices (try to choose good branching variables and good first branches).

Hint: The number of negative and positive occurrences of the variables $1, \dots, 5$ in F_2 are $(3, 2), (5, 4), (9, 3), (4, 7), (4, 4)$. **[10 marks]**

Question 2 Clause-sets, the DP-procedure and hitting clause-sets

(a) Basic notions

- (i) Define variables (\mathcal{VA}), literals (\mathcal{LIT}), clauses (\mathcal{CL}) and clause-sets (\mathcal{CLS}), and explain the meaning of these concepts. [4 marks]
- (ii) Compute the number of elements in $\mathcal{CLS}(\{1, \dots, n\})$ for $n \in \mathbb{N}_0$, that is, compute the number of clause-sets using n fixed variables. Explain your computation. [4 marks]
- (iii) Explain the correspondence between clauses and partial assignments, that is, explain how to create a clause C_φ out of a partial assignment φ , how to create a partial assignment φ_C out of a clause C , and what is the meaning of this correspondence. [4 marks]

(b) The DP-procedure

- (i) Explain how to compute $\text{DP}_v(F)$ for clause-sets F and variables v . [3 marks]
- (ii) Solve the SAT problem for

$$F = \{ \{ \bar{a}, e \}, \{ b, c, \bar{d} \}, \{ d, \bar{f} \}, \{ d, \bar{e} \}, \{ b, \bar{e} \}, \{ a, d, f \}, \{ \bar{b}, \bar{d} \} \}.$$

by applying the DP-procedure, showing the different stages of the computation. Hint: The amount of work needed for the DP-procedure depends on the order of the variables; by choosing a suitable order you can make your life easier. [4 marks]

- (c) Explain how to solve the SAT problem for hitting clause-sets efficiently, and why it works. [6 marks]

Question 3

(a) Graph colouring

(i) Define when a graph is “ k -colourable”. [1 marks]

(ii) Describe a general encoding of the graph k -colourability problem as a satisfiability problem. [5 marks]

(b) Partial assignments and autarkies

(i) Explain how $\varphi * F$ is computed. [2 marks]

(ii) Define when a partial assignment φ is an autarky for a clause-set F . [2 marks]

(iii) Prove that if φ is an autarky for F then $\varphi * F$ is satisfiability equivalent to F . [4 marks]

(c) Explain in detail how fast unit-clause propagation works, and argue why it is a linear time procedure. [11 marks]