CS_M42 2004/05
Constraint Satisfaction Problems and Applications

(Attempt 2 questions out of 3)

Question 1 SAT algorithms

(a) Discuss the trivial SAT algorithm (enumerating all total assignments), what it can achieve and how it can be improved. **[10 marks]**

(b) DLL-like SAT algorithms

(i) Explain the improved DLL algorithm scheme and the possible choices made in the computation. **[5 marks]**

(ii) Decide satisfiability of the clause-sets F_1, F_2 by using the improved DLL algorithm, where

$$F_1 := \big\{ \{c, b, \overline{e}\}, \{\overline{c}, d, a\}, \{\overline{b}, g\}, \{\overline{d}, f\}, \{c, \overline{b}, \overline{g}\}, \{\overline{c}, \overline{d}, \overline{f}\}, \{\overline{d}, f\}, \{d, \overline{a}\}, \{b, e\} \big\},$$

while F_2 is given in DIMACS format as

```
p cnf 5 15
-1 -3 4 0
-3 4 5 0
-2 -3 4 0
-1 -4 -5 0
-2 -4 -5 0
2 -4 5 0
-1 -2 3 0
2 3 4 0
3 4 -5 0
1 -3 -5 0
2 -3 4 0
-2 -3 -4 0
-3 4 5 0
2 -3 5 0
1 -2 -3 0
```

Show the backtracking trees and explain all your choices (try to chose good branching variables and good first branches).

Hint: The number of negative and positive occurrences of the variables $1, \ldots, 5$ in F_2 are $(3, 2), (5, 4), (9, 3), (4, 7), (4, 4)$. **[10 marks]**

Question 2 Clause-sets, the DP-procedure and hitting clause-sets

(a) Basic notions

(i) Define variables ($\mathcal{VA}$), literals ($\mathcal{LIT}$), clauses ($\mathcal{CL}$) and clause-sets ($\mathcal{CLS}$), and explain the meaning of these concepts. **[4 marks]**

(ii) Compute the number of elements in $\mathcal{CLS}(\{1,\dots,n\})$ for $n \in \mathbb{N}_0$, that is, compute the number of clause-sets using n fixed variables. Explain your computation. **[4 marks]**

(iii) Explain the correspondence between clauses and partial assignments, that is, explain how to create a clause C_φ out of a partial assignment φ, how to create a partial assignment φ_C out of a clause C, and what is the meaning of this correspondence. **[4 marks]**

(b) The DP-procedure

(i) Explain how to compute $\mathrm{DP}_v(F)$ for clause-sets F and variables v. **[3 marks]**

(ii) Solve the SAT problem for

$$F = \big\{ \{\overline{a}, e\}, \{b, c, \overline{d}\}, \{d, \overline{f}\}, \{d, \overline{e}\}, \{b, \overline{c}\}, \{a, d, f\}, \{\overline{b}, \overline{d}\} \big\}.$$

by applying the DP-procedure, showing the different stages of the computation. Hint: The amount of work needed for the DP-procedure depends on the order of the variables; by choosing a suitable order you can make your life easier. **[4 marks]**

(c) Explain how to solve the SAT problem for hitting clause-sets efficiently, and why it works. **[6 marks]**

Question 3

(a) Graph colouring

(i) Define when a graph is "k-colourable". **[1 marks]**

(ii) Describe a general encoding of the graph k-colourability problem as a satisfiability problem. **[5 marks]**

(b) Partial assignments and autarkies

(i) Explain how $\varphi * F$ is computed. **[2 marks]**

(ii) Define when a partial assignment φ is an autarky for a clause-set F. **[2 marks]**

(iii) Prove that if φ is an autarky for F then $\varphi * F$ is satisfiability equivalent to F. **[4 marks]**

(c) Explain in detail how fast unit-clause propagation works, and argue why it is a linear time procedure. **[11 marks]**