

PRIFYSGOL CYMRU; UNIVERSITY OF WALES

DEGREE EXAMINATIONS JANUARY 2002

SWANSEA

Computer Science

CS 216 Theory of Programming Languages

EXTERNAL CANDIDATES

Attempt 2 questions out of 3

Time allowed: 2 hours

Students are permitted to use the dictionaries provided by the University through the invigilators

January 2002 (External candidates)

CS _216. THEORY OF PROGRAMMING LANGUAGES

(Attempt 2 out of 3 questions)

Question 1

- (a) What is a *formal language* L ? Define the *recognition problem* for the language L . Define the mathematical concept of a *grammar* G and how it defines a formal language $L(G)$. [5 marks]
- (b) Explain how grammars are used in the definition of programming languages. What are modular grammars and how can they assist in the definition of programming languages? [5 marks]
- (c) Define the concept of a *context-free grammar*. State the *Pumping Lemma* for context free grammars. Give an example of a property of a programming language that cannot be defined by any context-free grammar. Sketch the mathematical method used to prove such facts. [10 marks]
- (d) Give a context-free grammar that defines *one* of the following
- (i) postal addresses for a country of your choice;
 - (ii) file name formats for an operating system of your choice;
 - (iii) addresses for a type of URL of your choice.
- [5 marks]

Question 2

- (a) Define carefully the mathematical concept of a (*many sorted*) *signature*. What aspect of a data type does a signature model? [4 marks]

- (b) Give a context-free grammar that defines the syntax of a language for defining all signatures. [5 marks]
- (c) *Sketch briefly* how to define the input-output semantics of the **while** language over all signatures. [9 marks]
- (d) What is a stream? What are *interactive systems* and what role does the data type of streams play in their modelling?
- Let Σ be any signature containing the booleans and natural numbers. Given a Σ -algebra A , construct a Σ_{Stream} -algebra A_{Stream} of infinite streams over A . [5 marks]
- (e) *Sketch briefly* how to define the semantics of a **while** language *with streams* over all signatures. [2 marks]

Question 3

- (a) Let Σ be a many sorted signature. Define carefully the concept of a (*many sorted*) Σ -algebra. What aspect of a data type does an algebra model? [4 marks]
- (b) Give a signature Σ for the integers and an example of a finite Σ algebra. Does your example satisfy the same laws as the standard model of the integers? [5 marks]
- (c) Let A and B be Σ -algebras. Define carefully the concept of a
- (i) Σ -homomorphism $f: A \rightarrow B$ and
 - (ii) Σ -isomorphism $f: A \rightarrow B$.

Briefly, explain their role in the theory of data types.

[6 marks]

- (d) Consider the following signature.

signature Machine;
sorts state, input;
operations next: state \times input \rightarrow state;
 output: state \times input \rightarrow output
endsig

Let M and N be two $\mathcal{M}_{\text{Machine}}$ algebras modelling two machines. Let $\varphi: M \rightarrow N$ be a $\mathcal{M}_{\text{Machine}}$ homomorphism. Write down the homomorphism equations for φ .

[4 marks]

- (e) Give Dedekind's axiomatic specification (\mathbb{N}, T) of the natural numbers. Explain how it captures precisely the abstract data type of natural numbers.

[6 marks]