PRIFYSGOL CYMRU; UNIVERSITY OF WALES

DEGREE EXAMINATIONS MAY/JUNE 2003

SWANSEA

Computer Science

CS 372 Numerical Algorithms and Computation

Attempt 2 questions out of 3

Time allowed: 2 hours

Students are permitted to use the dictionaries provided by the University

Students are permitted to use the calculators provided by the University

Numerical Algorithms and Computation
CS 372

*Answer **2** questions from **3**.*

1.a) By appealing to a *linear* spline and a *cubic* spline, explain what is meant by a spline interpolation.

[**1 marks**]

You are given a function B(x), defined over the five knots x = -2, -1, 0, 1, 2, with values B(-2)=B(+2)=0, B(-1)=B(+1)=1, B(0) =4, where

$$
B(x) \begin{cases} = 0 \text{ for } x \le -2 \text{ and } 2 \le x; & \\ = (x+2)^3 & \text{for } -2 \le x \le -1 \\ = 1 + 3(x+1) + 3(x+1)^2 - 3(x+1)^3 & \text{for } -1 \le x \le 0 \\ = 1 + 3(1-x) + 3(1-x)^2 - 3(1-x)^3 & \text{for } 0 \le x \le 1 \\ = (2-x)^3 & \text{for } 1 \le x \le 2 \end{cases}
$$

Show that B(x) is a cubic spline function.

Hence, or otherwise, qualify the number and type of conditions necessary to define a cubic spline. Comment upon the need for extra side conditions and upon common choices made, such as that leading to natural splines.

Plot the B(x) function and comment on its shape and properties outside the interval $-2 \le x \le 2$.

[**6 marks**]

b) Through polynomials of type $B(x)$, a general function $f(x)$ may be expressed at the knots, x_k, $0 \le k \le n$, as

$$f(x_k) = c_{k-1} + 4c_k + c_{k+1}$$

Write this system in matrix form for the coefficients, c_k, $0 \le k \le n$.

[**2 marks**]

For the natural spline choice, you are told that,

$$c_{-1} = 2c_0 - c_1 \quad \text{and} \quad c_{n+1} = 2c_n - c_{n-1}$$

Taking $n=4$, interval $[x_0, x_4]=[0, 0.5]$, $(x_k - x_{k-1})=h=0.5/n$, amend the system above for the first ($k=0$) and last ($k=n$) equations, assuming you are given $f(x_0)=0$, $f(x_n)=1$ and $f(x_k)=\sin \pi x_k$.

[**4 marks**]

c) Show in algorithmic steps, evaluation and pseudo-code how you would solve such a system of equations by Gauss-Seidel (GS) iteration for coefficients $\{c_1, c_2, c_3\}$, indicating two iterative sweeps and starting from a zero initial guess.

[**9 marks**]

Show what changes would be made to your GS-algorithm if Jacobi iteration were invoked instead.

[**1 marks**]

Comment upon the expected likelihood of convergence of this system and the differences anticipated between these iteration schemes.

[**2 marks**]

2.a) Define the Lagrange polynomial of degree two ($p_2(x)$), that interpolates the function $f(x)$ at three equally spaced points $x_0=a$, $x_1=c=(b+a)/2$, $x_2=b$, assuming $h=(b-a)/2=b-c=c-a$.

[**3 marks**]

Hence, or otherwise, construct Simpson's quadrature rule, based on function values $f(a)$, $f(c)$, $f(b)$.

[**5 marks**]

b) Apply Simpson's rule to the function $f(x)=0.5 + \sin \pi x$, over the interval $[1/4, 5/4]$, and compare your result to 6 sig. figs. to the analytic result 0.95016 in relative percentage error.

[**5 marks**]

c) Considering *pairs* of subintevals, establish the *composite* equivalent rule for $n/2$ pairs, where each (*i*th) pair is of total width $2h=0.5(x_{2i} - x_{2i-2})$, adopting uniform mesh-point spacing.

[**6 marks**]

For two pairs of subintervals ($n=4$), apply this rule again to the function $f(x)=0.5 + \sin \pi x$ on $[1/4, 5/4]$. Contrast your result against that in (2b).

[**6 marks**]

3.a) Describe the different types of error that are involved with the computer solution of ODEs? What is the meaning of local and global error?

[3 marks]

How may the error in a particular method be reduced?

[2 marks]

b) Starting from x=0 with a uniform stepsize of h=0.1, solve the following differential equation in ten steps to 4 d.p. using Euler's method,

$$y'(x) = x/y, \quad y(0) = 1.0, \; 0 \le x \le 1.0$$

to yield y(1.0) at the end of the interval. Compare your solution for quality against the analytical solution

$$y^2 = 1 + x^2$$

[7 marks]

Obtain a second estimate of y(1), by repeating the above procedure with double the stepsize, h=0.2. Comment upon the results obtained.

[6 marks]

c) Compare and contrast both Modified Euler (Improved Polygon) and Heun (predictor-corrector) methods, and provide pseudo-code to implement these algorithms.

[7 marks]