PRIFYSGOL CYMRU; UNIVERSITY OF WALES

DEGREE EXAMINATIONS MAY/JUNE 2002

SWANSEA

Computer Science

CS 372 Numerical Algorithms and Computation

Attempt 2 questions out of 3

Time allowed: 2 hours

Students are permitted to use the dictionaries provided by the University

Students are permitted to use the calculators provided by the University

Numerical Algorithms and Computation
CS 372

*Answer **2** questions from **3**.*

1.a) With the aid of diagrams, describe LU decomposition via the Crout algorithm. In this manner, or otherwise, outline the relationship between Gaussian elimination and LU decomposition.

[**5 marks**]

Provide pseudo-code for and implement the Crout algorithm, to calculate L and U factors, on the linear algebraic system below, of three equations in three unknowns (hint: retain rational numbers in your calculations),

$$\begin{aligned} 4x_1 + x_2 &= 6 \\ x_1 + 4x_2 + x_3 &= 12 \\ x_2 + x_3 &= 14 \end{aligned}$$

[**12 marks**]

b) In contrast, write pseudo-code to perform Gaussian Elimination, without pivoting and scaling.

Why are pivoting and scaling important, and where would they be incorporated into the standard algorithm?

Illustrate pivoting, by working step by step, down the first column of the following example:

$$\begin{aligned} x_2 + x_3 &= 14 \\ x_1 + 4x_2 + x_3 &= 12 \\ 4x_1 + x_2 &= 6 \end{aligned}$$

[**8 marks**]

2.a) Given the **data set A,** with 4 data points x_i and function values $f_0(x_i)$:

i	x_i	$f_0(x_i)$
0	1	0
1	2	1
2	4	2
3	1/2	-1

establish recurrence relationships and values for Newton divided-differences, $f_k(x_i)$, $k>0$, through and including, third-order differences (hint: retain rational numbers in your calculations). On this basis, define the third-order Newton interpolating polynomial and relate this to your table of difference values. Hence, or otherwise, evaluate the interpolating polynomial at $x=1.5$, providing your answer to two decimal places.

[**13 marks**]

b) Derive the equivalent Lagrange interpolation polynomial for **data set A** and explain why this replicates the Newton result above. Comment on how Newton and Lagrange polynomials differ in their functional representations and dependencies upon data points and function values, and in their efficiency of evaluation.

[**12 marks**]

3.a) State how many data points are required and what order of polynomial function is approximated exactly with the trapezium quadrature rule on a single interval? Explain why this is so and demonstrate this rule in diagrammatic form.

[**6 marks**]

b) Discuss the extension of the trapezium rule to Simpson's 1/3 rule by deriving this rule from first principles. Appealing to error estimates, comment upon the relative merits of this implementation over the Trapezium rule.

[**9 marks**]

c) Show how the Trapezium rule may be applied over n multiple, equal sized intervals (width h), deriving appropriate formulae. State what benefits this might bring and express this multiple segment implementation in pseudo-code.

[**10 marks**]