CS_116 (2004-2005)
MODELLING COMPUTING SYSTEMS
(*Attempt 2 questions out of 3*)

Question 1

In the game of **NIM**, an arbitrary number of piles of coins are formed, each with an arbitrary number of coins in them, and two players alternate in removing one or more coins from any one pile. The player who takes the last coin is declared to be the winner.

(a) Explain carefully how you can determine who has the winning strategy in NIM, and which moves are the winning moves. Use examples, and make it clear why the procedure works.
[8 marks]

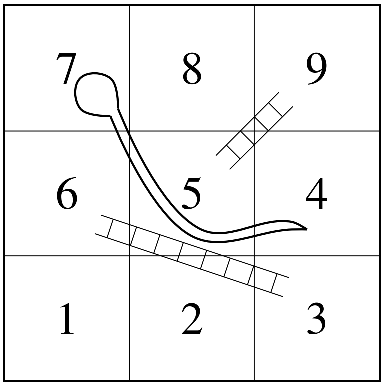
(b) Without referring to the above general theory of NIM, argue that in 2-pile NIM:

- the 2nd player has the winning strategy if the piles contain an *equal* number of coins;
- the 1st player has the winning strategy if the piles contain an *unequal* number of coins.

[4 marks]

(c) Who has the winning strategy in NIM when you start with n piles each containing the same number of coins? Explain your answer, again without referring to the general theory of NIM.
[3 marks]

The picture to the right shows a very simple variant of the children's board game SNAKES AND LADDERS. In this game, a single shared counter is started on square 1, and two players take turns moving the counter either ***one*** or ***two*** spaces forward. If the counter lands at the foot of a ladder, it climbs to the top of the ladder; and if the counter lands on the head of a snake, it slides down to the tail of the snake. The object of this game is to be the one to move the counter to the final square number 9.



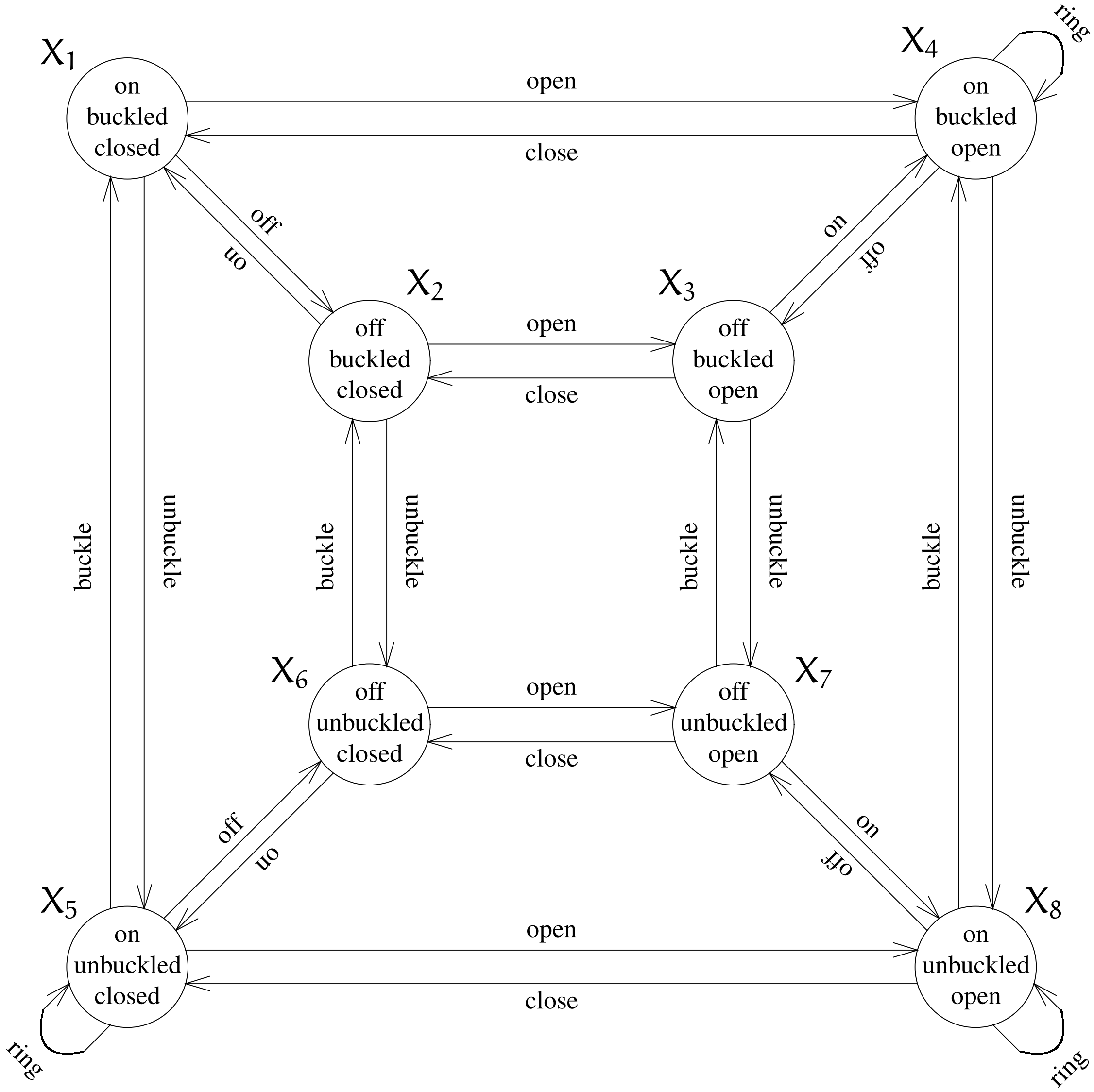
(d) List the positions in this game. (Note that these are not the same as the squares, because of the existence of the snakes and ladders.)
[2 marks]

(e) Identify which of the positions are winning positions, and which are losing positions; for the winning positions, indicate the winning move(s). (A losing position is one from which every move leads to a winning position, whereas a winning position is one from which there is a move to a losing position. Thus for example, 9 is a losing position, while 8 is a winning position with moving one space forward being the winning move.)
[8 marks]

Question 2

In this question, we study the specification of a car safety system, in which a bell rings (repeatedly) whenever the ignition is on while the door is open or the seatbelt is unbuckled.

The labelled transition system for this specification may be pictured as follows.



Here we have a system with

- eight states $\mathsf{S} = \left\{ \mathsf{X}_1, \mathsf{X}_2, \mathsf{X}_3, \mathsf{X}_4, \mathsf{X}_5, \mathsf{X}_6, \mathsf{X}_7, \mathsf{X}_8 \right\}$, and
- seven actions $A = \left\{ \text{open}, \text{close}, \text{buckle}, \text{unbuckle}, \text{on}, \text{off}, \text{ring} \right\}$.

For example, in state X_4, the ignition is on, the seatbelt is buckled, the door is open, and the alarm is ringing.

(a) The eight states in S can be given process definitions, such as

$$\mathsf{X}_1 \stackrel{\text{def}}{=} \text{off}.\mathsf{X}_2 \; + \; \text{open}.\mathsf{X}_4 \; + \; \text{unbuckle}.\mathsf{X}_5$$

Give such a definition for each of the state variables in S.

[4 marks]

(b) Let $\mathsf{D}(x)$, $\mathsf{F}(x)$, $\mathsf{M}(x)$ and $\mathsf{R}(x)$ be predicates defined over the states S as follows:

$$\begin{aligned} \mathsf{D}(x) &= \text{"the door is open in state } x\text{."} \\ \mathsf{F}(x) &= \text{"the seatbelt is buckled in state } x\text{."} \\ \mathsf{M}(x) &= \text{"the ignition is on in state } x\text{."} \\ \mathsf{R}(x) &= \text{"the bell is ringing in state } x\text{."} \end{aligned}$$

For each of these four predicates, indicate the states for which they are true.

[4 marks]

(c) Express $\mathsf{R}(x)$ in the modal logic $\mathcal{M}$ in two ways:

(i) one way involving only the action “ring”; and

(ii) another way *not* involving the action “ring”.
(Hint: First express $\mathsf{R}(x)$ in terms of $\mathsf{D}(x)$, $\mathsf{F}(x)$ and $\mathsf{M}(x)$.)

[5 marks]

(d) Which states satisfy the following formulas?

(i) $\langle \text{buckle} \rangle \mathit{true} \;\wedge\; \langle \text{close} \rangle \mathit{true}$

(ii) $\langle \text{buckle} \rangle \mathit{true} \;\wedge\; [\text{close}] \mathit{false}$

(iii) $\langle \text{on} \rangle \langle \text{ring} \rangle \mathit{true}$

(iv) $[\text{on}] \langle \text{ring} \rangle \mathit{true}$

(v) $\langle \text{open} \rangle \Big(\langle \text{buckle} \rangle \mathit{true} \;\wedge\; \langle \text{off} \rangle \mathit{true} \Big)$

(vi) $\langle \text{open} \rangle \Big(\langle \text{buckle} \rangle \mathit{true} \;\vee\; \langle \text{off} \rangle \mathit{true} \Big)$

[12 marks]

Question 3

(a) Consider the following process definition.

$$\mathsf{X} \stackrel{\text{def}}{=} a.\mathbf{0} + a.\mathsf{Z} \qquad \mathsf{Y} \stackrel{\text{def}}{=} a.\mathsf{Z} \qquad \mathsf{Z} \stackrel{\text{def}}{=} a.\mathsf{Z}$$

(i) Draw the labelled transition system for the above process.

[4 marks]

(ii) Explain in words how states X and Y differ, behaviourally.

[2 marks]

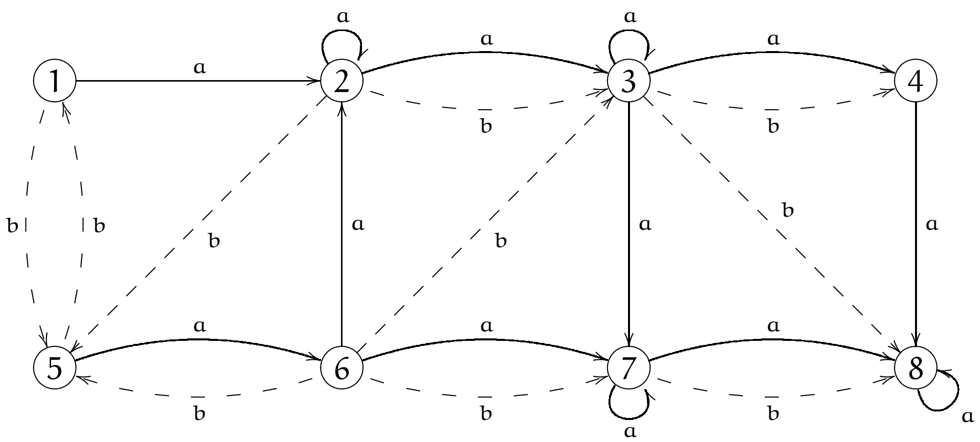
(iii) Give a modal formula which is true of state X but not true of state Y.

[2 marks]

(iv) Give a modal formula which is true of state Y but not true of state X.

[2 marks]

(b) Consider the following labelled transition system. (For ease of reading, all b-transitions are drawn using dashed lines.)



(i) Which states are 2-game equivalent to state 6? Explain why they are and the others are not.

[5 marks]

(ii) Which states are 2-game equivalent, but *not* 3-game equivalent, to state 6? Explain why they are not 3-game equivalent.

[5 marks]

(iii) Which states are n-game equivalent to state 5 for all n? Justify your answer.

[5 marks]