PRIFYSGOL CYMRU; UNIVERSITY OF WALES

DEGREE EXAMINATIONS JANUARY 2003

SWANSEA

Computer Science

CS 216 Theory of Programming Languages

(special version for dyslexic candidates)

Attempt 2 questions out of 3

Time allowed: 2 hours

Students are permitted to use the dictionaries provided by the University

Students are NOT permitted to use calculators

January 2003 (Special Formulation)

CS _216. THEORY OF PROGRAMMING LANGUAGES

Attempt TWO of the following three questions

Question 1

(a) Define carefully the mathematical concepts of a
(i) *many sorted signature* Σ.
(ii) *many sorted* Σ *algebra* A
What aspects of a data type do these two concepts model?

(**6 marks**)

(b) What is an axiomatic specification for a data type and what is its purpose?

Give Dedekind's axiomatic specification (Σ_D, T_D) of the natural numbers. Explain how it specifies the abstract data type of natural numbers.

(**8 marks**)

(c) Define the axiomatic specification (Σ_R, T_R) of a *commutative ring*. Give an example of a finite algebra that satisfies the axioms of a commutative ring.

(**7 marks**)

(d) Which of the following properties can or cannot be proved from the axioms of a commutative ring?

(i) $(\forall x, y)[(x-1)(x-2) = x^2 - 3x + 2]$
(ii) $(\forall x, y)[x.y = 0$ implies $x=0$ or $y=0]$

Give reasons for your answer.

(**4 marks**)

Question 2

(a) Let A and B be Σ-algebras. Define carefully the concept of a
(i) Σ-*homomorphism* ϕ: A → B and
(ii) Σ-*isomorphism* ϕ: A → B.
What are these concepts used for in the theory of abstract data types?

(**6 marks**)

(b) Let **Z** be the set of integers and let (**Z**, 0, +) be the algebra of integer addition. Which of the following functions f: (**Z**, 0, +) → (**Z**, 0, +) is a homomorphism? Give reasons for your answer.

(i) f(x) = 5x
(ii) f(x) = 5 + x
(iii) f(x) = 0

(**6 marks**)

(c) Consider the following signature which models a data storage medium:

signature Storage;
sorts state, address, data;
operations in: data × address × state → state;
out: address × state → data
endsig

Let M and N be two $\Sigma_{Storage}$ algebras modelling two storage systems. Let Φ: M → N be a $\Sigma_{Storage}$ homomorphism. Write down the two homomorphism equations for Φ.

(**4 marks**)

(d) What makes data digital and hence representable by a digital computer? Show how to model the idea of a digital data type using a homomorphism.

(**9 marks**)

Question 3

(a) What is a *formal language* L? Define the *recognition problem* for the language L. Define the mathematical concept of a *grammar G* and how it defines a formal language L(G).

(**6 marks**)

(b) The language of all signatures with **import** is a hierarchical and modular interface definition language for data types. Give a grammar that defines the language of all signatures with **import**.

(**6 marks**)

(c) The language of signatures *with* **import** is an extension to the simpler language of all signatures (*without* **import**). *Sketch briefly* how *flattening* may be used to define a semantics for this language.

(**4 marks**)

(d) *Sketch briefly* how to define the input-output semantics of the **while** language over all signatures with **import**.

(**9 marks**)