PRIFYSGOL CYMRU; UNIVERSITY OF WALES

DEGREE EXAMINATIONS JANUARY 2003

SWANSEA

Computer Science

CS 216 Theory of Programming Languages

external candidate

Attempt 2 questions out of 3

Time allowed: 2 hours

Students are permitted to use the dictionaries provided by the University

Students are NOT permitted to use calculators

January 2003 (External)

CS _216. THEORY OF PROGRAMMING LANGUAGES

Attempt TWO of the following three questions

Question 1

(a) What is a *formal language* L? Define the *recognition problem* for the language L. Define the mathematical concept of a *grammar* G and how it defines a formal language $L(G)$ by sequences of 1-step reductions.

(**7 marks**)

(b) Explain how grammars are used in the definition of programming languages. What are modular grammars and how can they assist in the definition of programming languages?

(**5 marks**)

(c) Define the concept of a *context-free grammar*. Give an example of a property of a programming language that cannot be defined by any context-free grammar. Sketch the mathematical method used to prove such facts.

(**8 marks**)

(d) Give a context-free grammar that defines postal addresses for a country of your choice.

(**5 marks**)

Question 2

(a) Define carefully the mathematical concept of a (*many sorted*) *signature*. What aspect of a data type does a signature model?

(**4 marks**)

(b) Give a context-free grammar that defines the syntax of a language for defining all signatures.

(**5 marks**)

(c) *Sketch briefly* how to define the input-output semantics of the **while** language over all signatures.

(**7 marks**)

(d) Show how to extend the grammar for the language of signatures given in part (b) to define the language of signatures that can **import** other signatures by name from a library of signatures.

(**2 marks**)

Explain, using an example,

(i) How the **import** construct introduces a modular and hierarchical structure to signatures.

(2 marks)

(ii) How this modular and hierarchical structure is removed by *flattening*.

(2 marks)

(e) The **while** language over all signatures *with* **import** WP_1 is an example of a complex extension to the simpler kernel language of the **while** language over all signatures (*without* **import**) WP_0. *Sketch briefly* how flattening may be used to define a semantics for the language WP_1 from the semantics of WP_0 given in part (c).

(3 marks)

Question 3

(a) Let Σ be a many sorted signature. Define carefully the concept of a (*many sorted*) *Σ-algebra*. What aspect of a data type does an algebra model?

(4 marks)

(b) Give a signature Σ for the real numbers and a Σ–algebra of real numbers.

(5 marks)

(c) Let *A* and *B* be Σ-algebras. Define carefully the concept of a

(i) Σ-*homomorphism* $\phi: A \rightarrow B$ and

(ii) Σ-*isomorphism* $\phi: A \rightarrow B$.

Briefly, explain their role in the theory of data types.

(6 marks)

(d) Consider the following signature which models a data storage medium:

signature Storage;
sorts state, address, data;
operations in: data × address × state → state;
out: address × state → data
endsig

Let M and N be two Σ_{Storage} algebras modelling two storage systems. Let $\Phi: M \rightarrow N$ be a Σ_{Storage} homomorphism. Write down the two homomorphism equations for Φ.

(4 marks)

(e) Give Dedekind's axiomatic specification (Σ, T) of the natural numbers. Explain how it captures precisely the abstract data type of natural numbers.

(6 marks)