PRIFYSGOL CYMRU; UNIVERSITY OF WALES

DEGREE EXAMINATIONS JANUARY 2003

SWANSEA

Computer Science

CS 376   Programming with Abstract Data Types

Attempt 2 questions out of 3

Time allowed: 2 hours

Students are permitted to use the dictionaries provided by the University

Students are NOT permitted to use calculators

CS_376 PROGRAMMING WITH ABSTRACT DATA TYPES

(Attempt 2 questions out of 3)

Question 1.

(a) What is an abstract data type? When is an abstract data type called monomorphic?

[4 marks]

(b) (i) Define what it means for a Σ-algebra to be *initial in a class* $\mathcal{C}$ of Σ-algebras.

(ii) Explain the difference between a *loose specification* and an *initial specification*, and sketch how a model for an initial specification can be constructed.

[7 marks]

(c) Let SET be the algebra of *finite sets of natural numbers* containing

- sorts boole, nat and set,
- the constants T, F, 0 and emptyset,
- the successor operation succ and an equality test equal on natural numbers,
- the boolean operation or,
- an operation insert inserting a number into a set,
- an operation element testing whether a given number is in a given set.

(i) Produce an initial specification (including the signature) for the algebra SET.

(iii) Identify a minimal set of generators for the algebra SET ('minimal' means no proper subset generates SET). Is SET freely generated by these generators? Justify your answer.

[14 marks]

Question 2.

(a) Explain the difference between minimal, intuitionistic and classical logic. What can you say about the behavior of these logics with respect to program synthesis from proofs?

[6 marks]

(b) (i) Derive in minimal logic the formula $P \to \neg\neg P$.

(ii) Derive in minimal logic the formula $P \wedge (Q \vee R) \to (P \wedge Q) \vee (P \wedge R)$.

(iii) Derive in classical logic the formula $\neg \forall x\, P(x) \to \exists x\, \neg P(x)$.

(iv) Give the proof term corresponding to the proof you found in question 2 (b)(i).

[14 marks]

(c) Explain why the following 'derivation' is not correct:

$$\dfrac{\dfrac{\dfrac{}{0=0}\,\text{refl}}{\exists x\,(x=0)}\,\exists^+ \qquad \dfrac{\dfrac{\dfrac{x=0}{\forall x\,(x=0)}\,\forall^+}{x=0\to\forall x\,(x=0)}\,\to^+}{\forall x\,(x=0\to\forall x\,(x=0))}\,\forall^+}{\forall x\,(x=0)}\,\exists^-$$

[5 marks]

Question 3.

(a) Discuss the advantages and disadvantages of defining abstract data types in a functional and in an object-oriented programming language. What are the differences with respect to the addition of generators and observers?

[7 marks]

(b) Let nat be a sort, $0\colon \mathsf{nat}$ a constant, $+, *\colon \mathsf{nat}\times\mathsf{nat}\to\mathsf{nat}$ operations (both used in infix notation), and $x, y, z\colon \mathsf{nat}$ variables. Prove that the term rewriting system given by rules

$$x+0\mapsto x$$
$$x*0\mapsto 0$$
$$x*(y+z)\mapsto x*y+x*z$$

is

(i) confluent,

(ii) terminating.

[12 marks]

(c) Let Σ be the signature containing one sort nat, one constant 0 and two binary operations, $+$ and $*$. Let A be the Σ-algebra of natural numbers with 0, addition and multiplication, and let α be a variable assignments such that $\alpha(x)$ is even for every variable x.

Prove by induction on terms that $t^{A,\alpha}$ is even for every Σ-term t.

[6 marks]